Infrared behavior of the ghost-gluon vertex in Landau gauge Yang–Mills theory

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Abstract

A semi-perturbative calculation of the ghost-gluon vertex in Landau gauge Yang-Mills theory in four and three Euclidean space-time dimensions is presented. Non-perturbative gluon and ghost propagators are employed, which have previously been calculated from a truncated set of Dyson–Schwinger equations and which are in qualitative and quantitative agreement with corresponding lattice results. Our results for the ghost-gluon vertex show only relatively small deviations from the tree-level one in agreement with recent lattice data. In particular, we do not see any sign for a singular behavior of the ghost-gluon vertex in the infrared.

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I. INTRODUCTION

The infrared behavior of QCD Green functions is of fundamental interest. In addition, these functions provide an important input for many calculations in hadron physics, for recent reviews see *e.g.* [1, 2, 3]. As infrared singularities are anticipated for some of these Green functions, non-perturbative continuum methods are needed to complement the knowledge gained in lattice Monte-Carlo calculations. Studies using different techniques, such as Dyson-Schwinger equations (see *e.g.* [2, 4, 5] and references therein), renormalization group methods [6], stochastic quantization [7], and lattice Monte-Carlo calculations (see *e.g.* [8, 9, 10, 11, 12] and references therein) have provided an unified picture of the infrared behavior of propagators in Landau gauge QCD in recent years. In this context the propagator of the Faddeev–Popov ghosts is of special interest: In the Landau gauge this propagator is infrared enhanced and diverges more strongly than $1/k^2$ for $k^2 \to 0$. On the one hand, this reflects the Zwanziger-Gribov horizon condition [7, 13, 14]. On the other hand, in the Landau gauge it enforces the Kugo–Ojima confinement criterion [15, 16]. The accompanying infrared suppression of the gluon propagator relates to positivity violation for transverse gluons by imposing a cut in the gluon propagator [5]. This resolves an old puzzle already encountered in perturbation theory, which has led to the Oehme-Zimmermann superconvergence relations [17].

These Landau-gauge studies are complemented by similar ones in the Coulomb gauge. Also in this gauge the infrared behavior of propagators is related to the Gribov problem and confinement [18]. In addition, it has been shown that center vortices play a crucial role in the infrared enhancement of ghosts [19]. Thus, the following picture emerges: Degrees of freedom belonging to the indefinite-metric part of state space like the ghosts in Landau gauge or ghosts and Coulomb gluons in Coulomb gauge are infrared enhanced. This infrared enhancement is related directly to an effective cutoff at the first Gribov horizon [7, 14]. The corresponding "excitations" are confining in that they mediate long-range correlations. Transverse gluons, on the other hand, are confined by these modes. The infrared part of the transverse gluon propagator is strongly suppressed. Besides an intuitive picture of confinement, this also provides a formal line of reasoning: Violations of positivity remove these states from the S matrix. Based on the relation of this picture to center vortices [19] it seems natural to speculate about the importance of topological field configurations in this context.

Although the picture, emerging from different methods described above, is in itself consistent and thus convincing, it is not yet complete. Finite-volume effects prevent lattice calculations to explore the extreme infrared. Functional continuum-based methods on the other hand necessarily involve truncations and the related errors are hard to control. For the functional methods the Landau gauge is advantageous due to its non-renormalization of the ghost-gluon vertex [20, 21]. To all orders in perturbation theory, the Landau gauge ghost-gluon vertex does not develop a genuine ultraviolet divergence, and especially for vanishing incoming ghost momentum it stays bare. Furthermore, it has been argued that, in the extreme infrared, the gauge fixing term dominates over the Yang-Mills action [7]. Therefore, the infrared behavior of all Green functions is expected to be dominated by contributions involving ghosts. This hypothesis has been tested for the gluon and ghost propagators and has proven to be correct, thus alleviating very strongly the issue of truncation induced errors. At this point, a truly non-perturbative investigation of the ghost-gluon vertex has a twofold aim: First, it will add a further test of ghost dominance in the infrared. Second, and

more importantly, the result is crucial to assess the validity of recent investigations based on functional methods as all but the very first investigations[1] [22, 23] used a bare ghost-gluon vertex. Thus, we will present a semi-perturbative calculation of the ghost-gluon vertex based on its Dyson-Schwinger equation (DSE). For this project, it has proven advantageous that lattice results for the Landau gauge ghost-gluon vertex have been published very recently [24]. We will compare our predictions to these data.

This paper is organized as follows: To make it reasonably self-contained we will briefly discuss the non-perturbative gluon- and ghost propagators as they emerge from the solutions of their DSE's, truncated at the level of propagators. Then, a truncation for the DSE of the ghost-gluon vertex will be given. We then discuss the results of a semi-perturbative evaluation of this vertex. To this end, two types of input for the vertex to be calculated are used. This, and the comparison to lattice results, provides strong evidence that the full, non-perturbative ghost-gluon vertex is very close to the tree-level one for all momenta.

II. GLUON- AND GHOST PROPAGATORS IN LANDAU GAUGE QCD

Yang-Mills theory in D-dimensional Euclidean spacetime in the Landau gauge is described by the Lagrangian [2]

$$\begin{aligned} \mathcal{L} = &= \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \\ F^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_D f^{abc} A^b_\mu A^c_\nu \\ D^{ab}_\mu &= \delta^{ab} \partial_\mu + g_D f^{abc} A^c_\mu \ , \end{aligned} \tag{1}$$

where $F^a_{\mu\nu}$ denotes the field strength tensor, D^{ab}_μ the covariant derivative, g_D the D-dimensional gauge coupling, and f^{abc} the structure constants of the gauge group. A^a_μ is the gluon field and $\bar{c}^a$ and c^a are the Faddeev-Popov ghost fields, describing part of the intermediate states of the gluon field.

Within this framework, in Euclidean momentum space the Landau gauge gluon and ghost propagators, $D_{\mu\nu}(p)$ and $D_G(p)$, can be generically written as

$$D_{\mu\nu}(p, \mu^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2, \mu^2)}{p^2} \ , \qquad D_G(p, \mu^2) = -\frac{G(p^2, \mu^2)}{p^2} \ , \tag{2}$$

where μ^2 denotes the renormalization scale, and $Z(p^2, \mu^2)$ and $G(p^2, \mu^2)$ are the gluon and ghost dressing functions. They can be determined from a solution of their DSE's [4, 25, 26] using a well-established truncation scheme [22, 25]. A recent comparison of these solutions to the corresponding lattice results can be found in ref. [27, 28]. In the infrared, *i.e.* for infinitesimally small p^2, these equations can be solved analytically [7, 21, 22] and one finds simple power laws,

$$Z(p^2, \mu^2) \sim (p^2)^{2\kappa + 2 - D/2} \ , \qquad G(p^2, \mu^2) \sim (p^2)^{-\kappa} \ , \tag{3}$$

for the gluon- and ghost dressing function with exponents related to each other and to the dimensionality D. Here κ is an irrational number, $\kappa \approx 0.595$ for $D = 4$ and $\kappa \approx 0.398$ for

[1] In these studies a ghost-gluon vertex which is an approximate solution to the corresponding Slavnov-Taylor identity has been employed.

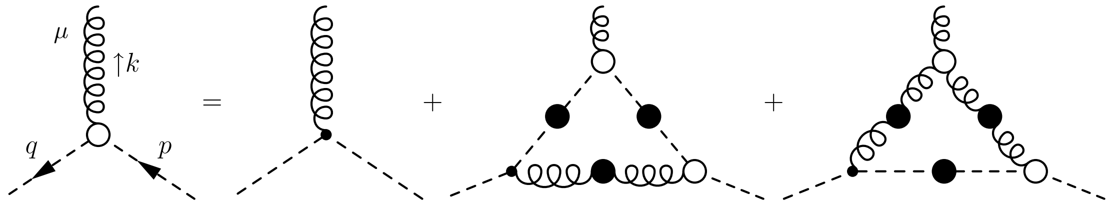

FIG. 1: The truncated DSE for the ghost-gluon vertex. Dotted lines denote ghosts, and wiggly lines gluons. Lines with a dot indicate full propagators. Vertices with small black dots represent bare vertices and open circles represent full vertices. Contributions from the ghost-gluon scattering kernel have been neglected.

$D = 3$ [21, 29]. These analytical results depend slightly on the truncation scheme[2] [21, 26]. As mentioned above, this is in agreement with the Kugo-Ojima confinement criterion and Zwanziger's horizon condition.

In four space-time dimensions the ghost- and gluon dressing functions can be used to define a non-perturbative running coupling [22]

$$\alpha(p^2) = \alpha(\mu^2)\, G^2(p^2, \mu^2)\, Z(p^2, \mu^2) \; . \tag{4}$$

Due to the ultraviolet finiteness of the ghost-gluon vertex in Landau gauge, no vertex function appears in this definition. Note that the r.h.s. of eq. (4) is a renormalization group invariant, and thus $\alpha(p^2)$ does not depend on the renormalization point. From the analytical power laws (3) one infers that the coupling has a fixed point in the infrared, given by $\alpha(0) \approx 8.92/N_c$. The infrared dominance of the ghosts imply that $\alpha(0)$ depends only weakly on the dressing of the ghost-gluon vertex and not at all on other vertex functions [21].

In the following, for the calculation of the ghost-gluon vertex in four space-time dimensions the pointwise accurate fit [25]

$$\begin{aligned} \alpha(p^2) &= \frac{\alpha(0)}{\ln(e + a_1 p^{2a_2} + b_1 p^{2b_2})} \; , \qquad & R(p^2) &= \frac{c p^{2\kappa} + d p^{4\kappa}}{1 + c p^{2\kappa} + d p^{4\kappa}} \; , \\ Z(p^2) &= \left(\frac{\alpha(p^2)}{\alpha(\mu^2)} \right)^{1+2\delta} R^2(p^2) \; , \qquad & G(p^2) &= \left(\frac{\alpha(p^2)}{\alpha(\mu^2)} \right)^{-\delta} R^{-1}(p^2) \; , \end{aligned} \tag{5}$$

will be used. It employs fitting parameters a_1, a_2, b_1, b_2 and c, d for the running coupling $\alpha(p^2)$ and the auxiliary function $R(p^2)$, respectively. Here, $\delta = -9/44$, is the anomalous dimension of the ghost dressing function and $\alpha(\mu^2 = (1.31\ \text{GeV})^2) = 0.9676$. The six parameters of the fit are given by $a_1 = 5.292\ \text{GeV}^{-2a_2}$, $a_2 = 2.324$, $b_1 = 0.034\ \text{GeV}^{-2b_2}$, $b_2 = 3.169$, $c = 1.8934\ \text{GeV}^{-2\kappa}$ and $d = 4.6944\ \text{GeV}^{-4\kappa}$. For the calculation in $D = 3$, the numerical results for $G(p^2)$ and $Z(p^2)$ [26] are directly used.

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[2] For $D = 4$ one can show, independent of any truncation, that $\kappa > 0$ [30].

III. GHOST-GLUON VERTEX

In the Landau gauge, the most general tensor structure of the ghost-gluon vertex with gluon momentum k and ghost momenta p and q is given by

$$\Gamma^{abc}_\mu(k;q,p) = ig_D \left(q_\mu \left(f^{abc} + A^{abc}(k^2;q^2,p^2) \right) + k_\mu B^{abc}(k^2;q^2,p^2) \right) \ , \tag{6}$$

where A^{abc} and B^{abc} are scalar functions describing the deviation from the tree-level form, and g_D is the coupling constant. As there is no indication for a color structure different from the one occurring in perturbation theory [2] we assume that $A^{abc}(k^2;q^2,p^2) =: f^{abc}A(k^2;q^2,p^2)$ and $B^{abc}(k^2;q^2,p^2) =: f^{abc}B(k^2;q^2,p^2)$. Note, that B is only relevant off-shell.

The complete DSE for the ghost-gluon vertex is derived in the appendix[3]. According to the truncation scheme adopted for the propagators [22, 25], the four-point function is neglected in the following. The truncated DSE is shown in Fig. 1 and is given in momentum space by

$$\begin{aligned}
\Gamma_\mu(k;q,p) \ =\ & \Gamma^{(0)}_\mu(k;q,p) \\
& -\frac{1}{2} g_D^2 N_c \int \frac{d^D\omega}{(2\pi)^D} \Gamma_\mu(k;\omega,\omega+k) \\
& \qquad \times D_G(\omega)\Gamma^{(0)}_\nu(q) D_{\nu\lambda}(\omega-q)\Gamma_\lambda(q-\omega;\omega+k,p) D_G(\omega+k) \\
& -\frac{1}{2} g_D^2 N_c \int \frac{d^D\omega}{(2\pi)^D} \Gamma_{\mu\nu\rho}(k,\omega,\omega+k) \\
& \qquad \times D_{\nu\lambda}(\omega)\Gamma^{(0)}_\lambda(q) D_G(\omega-q)\Gamma_\sigma(\omega+k;q-\omega,p) D_{\rho\sigma}(\omega+k) \ ,
\end{aligned} \tag{7}$$

where $\Gamma^{(0)}_\nu$ is the bare ghost-gluon vertex and $\Gamma_{\mu\nu\rho}$ the connected 3-gluon vertex. The momentum routing follows the same conventions as in ref. [2].

Although the ghost-gluon scattering kernel is neglected, a self-consistent solution of this equation, together with the propagator equations, is of significant technical complexity. Fortunately, as we will see below, such a procedure is not necessary. It is sufficient to perform a semi-perturbative calculation, *i.e.* to do one iteration step in the ghost-gluon vertex DSE. If our starting hypothesis is correct, the resulting vertex should not significantly deviate from the input tree-level vertex. As a further test, we will also employ as input an ansatz for the ghost-gluon vertex, which is an approximate solution of the corresponding Slavnov-Taylor identity [22],

$$\Gamma^{abc}_\mu(k;q,p) = ig_D f^{abc} q_\mu \left(\frac{G(k^2)}{G(q^2)} + \frac{G(k^2)}{G(p^2)} - 1 \right) \ . \tag{8}$$

First, we will present the results with the input vertices left bare and only the propagators dressed as described above. The results are displayed in Fig. 2 for the $D = 4$ case and in Fig. 3 for $D = 3$. Note that these functions have the proper ghost-antighost symmetry [21], *e.g.* $A(k^2;q^2,p^2) = A(k^2;p^2,q^2)$ [31]. The transverse part of the ghost-gluon vertex, $1 + A$, is extracted employing

$$\frac{k^2}{ig_D\Delta} q_\nu \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \Gamma^{abc}_\mu(k;q,p) = f^{abc}(1 + A(k^2;q^2,p^2)) \tag{9}$$

[3] The general structure of this DSE was already given in [32].

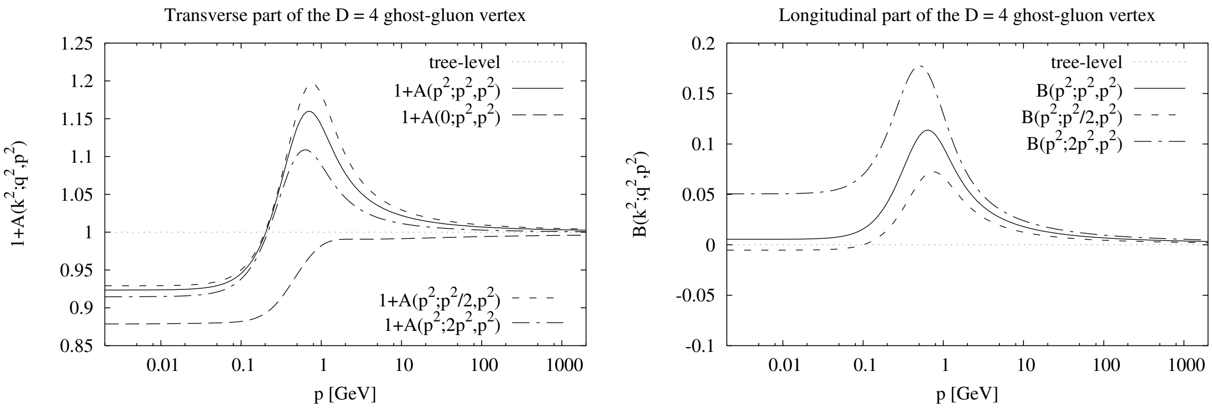

FIG. 2: The normalized transverse part $1 + A$ (left panel) and the normalized longitudinal part B (right panel) of the ghost-gluon vertex for $D = 4$ in various kinematical regions.

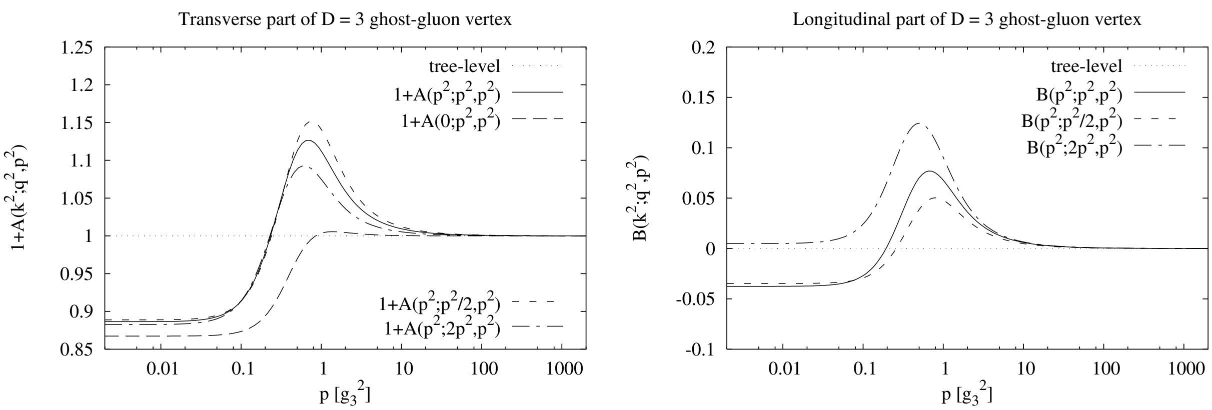

FIG. 3: Same as Fig. 2 for $D = 3$.

where $\Delta = q^2k^2 - (q \cdot k)^2$ is a Gram determinant. The deviations of the transverse part from tree-level are clearly less than 20%. This is true for all momenta allowed by momentum conservation [31]. In addition, also the longitudinal part, $B(k^2; q^2, p^2)$, is smaller than 0.2 for almost all momenta and finite everywhere.

Thus, our results indicate that the full self-consistent solution will likely be very close to the tree-level form. A crucial further test is provided, if the non-trivial form (8) is used as input on the r.h.s. of the the equation for the ghost-gluon vertex.[4] Several observations can be inferred from Fig. 4. First, also in this case deviations from tree-level are small, except for those kinematical regions where an infrared singularity is enforced by the ansatz (8). Second, and even more important, the calculated vertex function A is much closer to the tree-level case than the input. A systematic study of possible input vertex choices yields the same result [31]. Furthermore, for $D = 3$ the results are very similar to the $D = 4$ ones [31].

Finally, we want to compare our results to recent lattice results [24] in Fig. 5. These

[4] The 3-gluon vertex is taken bare. Recent investigations showed that the self-consistently determined infrared divergence of the 3-gluon vertex does not change the result presented here [33].

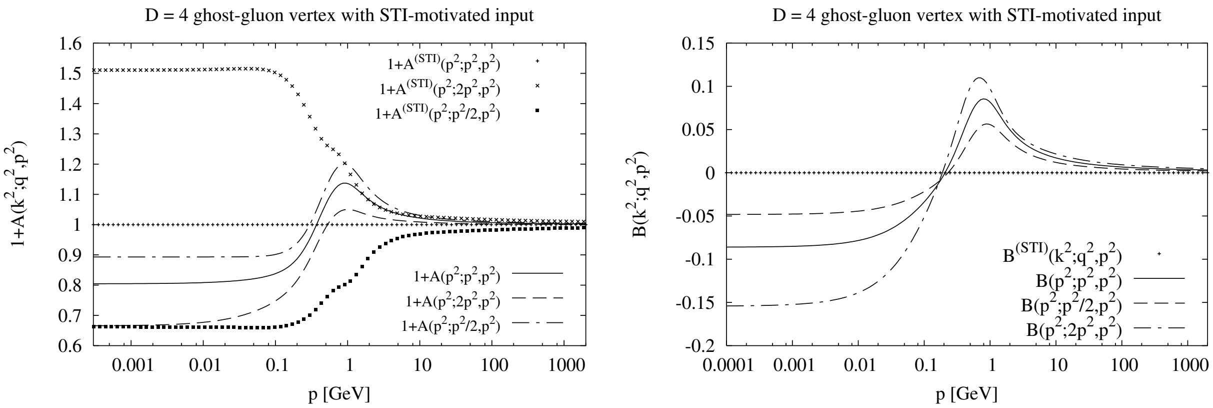

FIG. 4: Same as Fig. 2 but here with eq. (8) as input for the ghost-gluon vertex. The input vertices are denoted by (STI) and represented by crosses.

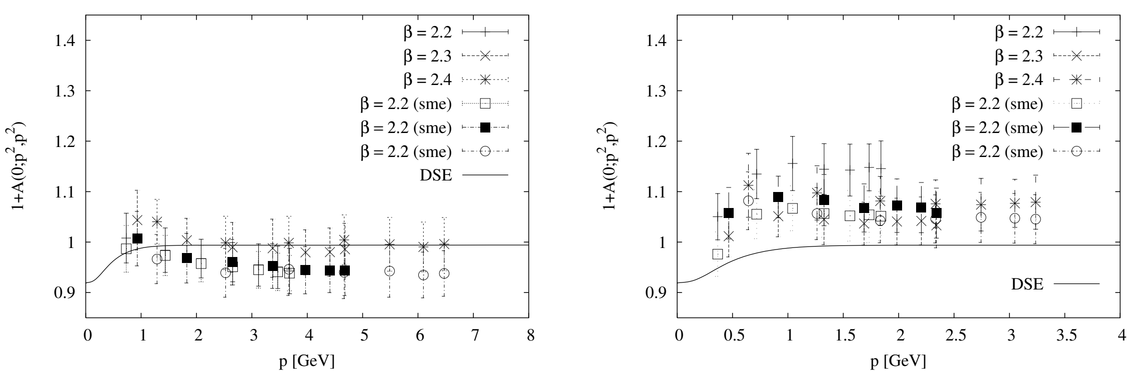

FIG. 5: The function $1 + A(0; p^2, p^2)$ for D=4 and two colors as compared to the corresponding lattice results [24]. The left panel shows the results obtained with symmetrically chosen momenta, the right panel with an asymmetric choice, see ref. [24] for further details.

calculations have been performed for gauge group SU(2). Thus, we change the color prefactor of the loop diagrams in Fig. 1 accordingly.[5] Also, in the lattice calculation only the ghost-gluon vertex for vanishing gluon momentum has been determined, *i.e.* in our notation $1 + A(0; p^2, p^2)$ has been calculated. Furthermore, the smallest momentum available on the lattice is 366 MeV, and this only at the expense of an asymmetrically chosen momentum, see ref. [24] for more details. Note that for symmetrically chosen momenta, see left panel of Fig. 5, the lattice results for the vertex are less affected by the breaking of rotational symmetry. Given the systematic error in the lattice calculation, one can conclude that the lattice results are, within errors, consistent with the tree-level form at all momenta considered. Our results nicely match this behavior. In addition, we predict a slight decrease at small momenta.

A feature of our results is the seemingly non-uniform limit for the functions A and B when all three momenta vanish. This is due to the ordering of the momenta when performing the limit. As these functions are finite, the full ghost-gluon vertex (including the corresponding

[5] This prefactor is $f^{abc} N_c/2$ for a general number of colors.

prefactors, see eq. (6)) is regular. In particular, for vanishing incoming ghost momentum, the ghost-gluon vertex is bare as expected [20, 21].

IV. CONCLUSIONS

We have presented approximate non-perturbative solutions for the ghost-gluon vertex in Landau gauge for Euclidean momenta in $D = 4$ and $D = 3$. To this end, we have employed non-perturbative gluon and ghost propagators. We used two types of input for the ghost-gluon vertex in the loop diagrams in order to estimate the behavior of a fully self-consistent solution. We have also compared our results to those of a recent lattice calculation.

These results, when taken together, show rather conclusively that deviations of the ghost-gluon vertex from its tree-level value are very small, especially in the infrared. They thus validate the truncation scheme used to calculate the propagators of the Yang-Mills theory. More importantly, they confirm the strong evidence for infrared ghost dominance in Landau gauge, and thus for the Zwanziger-Gribov scenario, as they fulfill Zwanziger's hypothesis of a bare ghost-gluon vertex in the infrared.

The results are not expected to change qualitatively when including matter fields, since the input propagators are rather insensitive [4, 26] to quark contributions and no additional terms appear in the truncated DSE. This is quite distinct from similar calculations for the quark-gluon vertex, which find significant deviations from the tree-level form, possibly involving infrared divergences [34].

In summary, the results presented here, nicely match a picture of confinement where the confining fields are on or near the Gribov horizon. They provide a further piece of evidence for a confinement mechanism of the Kugo-Ojima or Zwanziger-Gribov type.

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APPENDIX: DERIVATION OF THE DSE FOR THE GHOST-GLUON VERTEX

To derive the DSE for the ghost-gluon vertex it is only necessary to consider an action $\mathcal{S}_{gh}$ that involves the contributions from ghosts in the Lagrangian (1), although due to the mutual coupling of the Green functions, the entire Lagrangian is implicitly relevant. Furthermore, we introduce J^a_μ, $\bar{\sigma}^a$ and σ^a as sources for the fields A^a_μ, c^a and $\bar{c}^a$, respectively, so that we can define the generating functional for full Green functions,

$$Z[J,\bar{\sigma},\sigma] = \int \mathcal{D}[A\bar{c}c] \exp\left(-\int d^D x \mathcal{L} + \int d^D x \left(A^a_\mu J^a_\mu + \bar{\sigma}^a c^a + \bar{c}^a \sigma^a\right)\right), \qquad \text{(A1)}$$

the generating functional for connected Green functions, $W[J,\bar\sigma,\sigma] = \ln Z$, and the one for proper Green functions, $\Gamma[A,c,\bar c] = -W[J,\bar\sigma,\sigma] + \int d^dx(A^a_\mu J^a_\mu + \bar\sigma^a c^a + \bar c^a \sigma^a)$. The fields and sources are then given by

$$\begin{aligned} \frac{\delta W}{\delta \sigma^a} &= \bar c^a, \qquad & \frac{\delta W}{\delta \bar\sigma^a} &= c^a, \qquad & \frac{\delta W}{\delta J^a_\mu} &= A^a_\mu, \\ \frac{\delta \Gamma}{\delta c^a} &= \bar\sigma^a, \qquad & \frac{\delta \Gamma}{\delta \bar c^a} &= \sigma^a, \qquad & \frac{\delta \Gamma}{\delta A^a_\mu} &= J^a_\mu, \end{aligned} \tag{A2}$$

where we use right derivatives for the following Grassmann fields,

$$\frac{\delta}{\delta c^a} := \frac{\overleftarrow{\delta}}{\delta c^a} \quad , \qquad \frac{\delta}{\delta \sigma^a} := \frac{\overleftarrow{\delta}}{\delta \sigma^a} \, . \tag{A3}$$

One way to approach the DSE for the ghost-gluon vertex is to start with the identity

$$\begin{aligned} 0 &= \int \mathcal{D}[A\bar c c] \frac{\delta}{\delta \bar c^b(y)} \exp\left(- \int d^D x \mathcal{L} + \int d^D x \left(A^a_\mu J^a_\mu + \bar\sigma^a c^a + \bar c^a \sigma^a \right) \right) \\ &= \left\langle - \frac{\delta \mathcal{S}_{gh}\left[A, c, \bar c\right]}{\delta \bar c^b(y)} + \sigma^b(y) \right\rangle \, . \end{aligned} \tag{A4}$$

The expression in the brackets represents a full correlation function as generated by Z. Retaining non-zero sources, we now apply to the above expression the derivative

$$\frac{\delta}{\delta c^c(z)} = \int d^D v \frac{\delta^2 \Gamma}{\delta \bar c^d(v) \delta c^c(z)} \frac{\delta}{\delta \sigma^d(v)} + \text{vanishing terms} \, . \tag{A5}$$

Some terms vanish because the functionals can depend on pairs of Grassmann fields only. Throughout the calculation, great care is mandatory when dropping terms since most of them vanish only when setting sources to zero. We define the ghost and gluon propagators in position space

$$\tilde D^{ab}_G(x-y) := \left\langle c^a(x) \bar c^b(y) \right\rangle\Big|_{\eta\equiv 0} = \left. \frac{\delta^2 W}{\delta \overline{\sigma}^a(x) \delta \sigma^b(y)} \right|_{\eta \equiv 0} , \tag{A6}$$

$$\tilde D^{ab}_{\mu\nu}(x-y) := \left\langle A^a_\mu(x) A^b_\nu(y) \right\rangle\Big|_{\eta\equiv 0} = \left. \frac{\delta^2 W}{\delta J^a_\mu(x) \delta J^b_\nu(y)} \right|_{\eta \equiv 0} , \tag{A7}$$

as well as the proper ghost-gluon vertex in position space

$$\tilde\Gamma^{abc}_\mu(x;y,z) := \left. \frac{\delta^3 \Gamma}{\delta A^a_\mu(x) \delta \overline{c}^b(y) \delta c^c(z)} \right|_{\eta\equiv 0} . \tag{A8}$$

After usage of the relation

$$\delta(x-y)\delta^{ab} = \frac{\delta \bar\sigma^b(y)}{\delta \bar\sigma^a(x)} = \int d^D z \frac{\delta \bar\sigma^b(y)}{\delta \bar c^d(z)} \frac{\delta \bar c^d(z)}{\delta \bar\sigma^a(x)} = \int d^D z \frac{\delta^2 \Gamma}{\delta \bar c^d(z) \delta c^b(y)} \frac{\delta^2 W}{\delta \overline{\sigma}^a(x) \delta \sigma^d(z)} \tag{A9}$$

one then obtains

$$\frac{\delta^2\Gamma}{\delta\overline{c}^b(y)\delta c^c(z)} Z[J,\overline{\sigma},\sigma] \;=\; \partial^2\delta^{bc}\delta(y-z)Z[J,\overline{\sigma},\sigma] \\ +g_D f^{bgh}\partial^y_\rho \int d^Dv \frac{\delta^2\Gamma}{\delta\overline{c}^d(v)\delta c^c(z)} \left\langle A^h_\rho(y)c^g(y)\overline{c}^d(v)\right\rangle \;. \quad \text{(A10)}$$

To find the DSE for the ghost-gluon vertex, we apply to equation (A10) the derivative

$$\frac{\delta}{\delta A^a_\mu(x)} = \int d^Du \frac{\delta^2\Gamma}{\delta A^a_\mu(x)\delta A^e_\nu(u)} \frac{\delta}{\delta J^e_\nu(u)} + \text{vanishing terms}\,. \quad \text{(A11)}$$

One can now immediately set sources to zero to find the proper ghost-gluon vertex:

$$\tilde{\Gamma}^{abc}_\mu(x;y,z) \;=\; g_D f^{bgh}\partial^y_\rho \int d^Dv\, \frac{\delta^3\Gamma}{\delta A^a_\mu(x)\delta\overline{c}^d(v)\delta c^c(z)} \left\langle A^h_\rho(y)c^g(y)\overline{c}^d(v)\right\rangle\Big|_{\eta\equiv 0} \\ +g_D f^{bgh}\partial^y_\rho \int d^D[uv]\, \frac{\delta^2\Gamma}{\delta\overline{c}^d(v)\delta c^c(z)} \frac{\delta^2\Gamma}{\delta A^a_\mu(x)\delta A^e_\nu(u)} \left\langle A^h_\rho(y)A^e_\nu(u)c^g(y)\overline{c}^d(v)\right\rangle\Big|_{\eta\equiv 0} \;. \quad \text{(A12)}$$

The decomposition of the full 4-point correlation function yields

$$\begin{aligned} &\left\langle A^h_\rho(y)A^e_\nu(u)c^g(y)\overline{c}^d(v)\right\rangle\Big|_{\eta\equiv 0} \\ &= \frac{\delta^2 W}{\delta J^h_\rho(y)\delta J^e_\nu(u)} \frac{\delta^2 W}{\delta\overline{\sigma}^g(y)\delta\sigma^d(v)}\Big|_{\eta\equiv 0} + \frac{\delta}{\delta J^h_\rho(y)} \frac{\delta^3 W}{\delta J^e_\nu(u)\delta\overline{\sigma}^g(y)\delta\sigma^d(v)}\Big|_{\eta\equiv 0} \\ &= \frac{\delta^2 W}{\delta J^h_\rho(y)\delta J^e_\nu(u)} \frac{\delta^2 W}{\delta\overline{\sigma}^g(y)\delta\sigma^d(v)}\Big|_{\eta\equiv 0} + \int d^D[rst]\times \\ &\Bigg\{ -\frac{\delta^2 W}{\delta J^e_\nu(u)\delta J^k_\lambda(r)} \frac{\delta^2 W}{\delta\overline{\sigma}^g(y)\delta\sigma^m(s)} \frac{\delta^3\Gamma}{\delta A^k_\lambda(r)\delta\overline{c}^m(s)\delta c^n(t)} \frac{\delta^3 W}{\delta J^h_\rho(y)\delta\overline{\sigma}^n(t)\delta\sigma^d(v)} \\ &- \frac{\delta^3 W}{\delta J^h_\rho(y)\delta J^e_\nu(u)\delta J^k_\lambda(r)} \frac{\delta^2 W}{\delta\overline{\sigma}^g(y)\delta\sigma^m(s)} \frac{\delta^3\Gamma}{\delta A^k_\lambda(r)\delta\overline{c}^m(s)\delta c^n(t)} \frac{\delta^2 W}{\delta\overline{\sigma}^n(t)\delta\sigma^d(v)} \\ &- \int d^Dw \frac{\delta^2 W}{\delta J^e_\nu(u)\delta J^k_\lambda(r)} \frac{\delta^2 W}{\delta J^h_\rho(y)\delta J^l_\sigma(w)} \frac{\delta^2 W}{\delta\overline{\sigma}^g(y)\delta\sigma^m(s)} \\ &\qquad\times \frac{\delta^4\Gamma}{\delta A^l_\sigma(w)\delta A^k_\lambda(r)\delta\overline{c}^m(s)\delta c^n(t)} \frac{\delta^2 W}{\delta\overline{\sigma}^n(t)\delta\sigma^d(v)} \\ &- \frac{\delta^2 W}{\delta J^e_\nu(u)\delta J^k_\lambda(r)} \frac{\delta^3 W}{\delta J^h_\rho(y)\delta\overline{\sigma}^g(y)\delta\sigma^m(s)} \frac{\delta^3\Gamma}{\delta A^k_\lambda(r)\delta\overline{c}^m(s)\delta c^n(t)} \frac{\delta^2 W}{\delta\overline{\sigma}^n(t)\delta\sigma^d(v)} \Bigg\}\Bigg|_{\eta\equiv 0} \,. \end{aligned} \quad \text{(A13)}$$

The last term of the last line in eq. (A13) produces a 3PI-graph which, however, cancels in eq. (A12) with the first term. We now introduce two further definitions. The proper three-gluon vertex shall be denoted by

$$\tilde{\Gamma}^{and}_{\mu\nu\lambda}(x,w,r) := \frac{\delta^3\Gamma}{\delta A^a_\mu(x)\delta A^n_\nu(w)\delta A^d_\lambda(r)}\Big|_{\eta\equiv 0} \;, \quad \text{(A14)}$$

and the proper 4-point Green function involving two gluons and two ghosts is defined by

$$\tilde{\Gamma}^{nagc}_{\sigma\mu}(w,x;t,z) := \left.\frac{\delta^4\Gamma}{\delta A^n_\sigma(w)\delta A^a_\mu(x)\delta\overline{c}^g(t)\delta c^c(z)}\right|_{\eta\equiv 0} . \tag{A15}$$

After further decompositions of connected into proper 3-point correlation functions and using (A9), eq. (A12) can be rewritten[6] as

$$\begin{aligned}\tilde{\Gamma}^{abc}_\mu(x;y,z) \;=\; & g_D f^{abc}\partial^y_\mu\delta(y-x)\delta(y-z)\\ & +g_D f^{hbm}\partial^y_\rho\int d^D[rstw]\tilde{D}^{hg}_{\rho\sigma}(y-t)\tilde{D}^{mn}_G(y-w)\tilde{\Gamma}^{and}_\mu(x;w,r)\tilde{D}^{de}_G(r-s)\tilde{\Gamma}^{gec}_\sigma(t;s,z)\\ & +g_D f^{mbh}\partial^y_\rho\int d^D[rstw]\tilde{D}^{hg}_G(y-t)\tilde{D}^{mn}_{\rho\nu}(y-w)\tilde{\Gamma}^{and}_{\mu\nu\lambda}(x,w,r)\tilde{D}^{de}_{\lambda\sigma}(r-s)\tilde{\Gamma}^{egc}_\sigma(s;t,z)\\ & -g_D f^{mbh}\partial^y_\rho\int d^D[tw]\tilde{D}^{hg}_G(y-t)\tilde{\Gamma}^{nagc}_{\sigma\mu}(w,x;t,z)\tilde{D}^{mn}_{\rho\sigma}(y-w)\,.\end{aligned} \tag{A16}$$

The last step to take is to identify the bare ghost-gluon vertex which is derived from the Lagrangian (1) as

$$\tilde{\Gamma}^{(0)abc}_\mu(x;y,z) := \frac{\delta^3\mathcal{S}_{gh}}{\delta A^a_\mu(x)\delta\overline{c}^b(y)\delta c^c(z)} = g_D f^{abc}\partial^y_\mu\delta(y-x)\delta(y-z)\,. \tag{A17}$$

Thus one readily obtains

$$g_D f^{hbm}\partial^y_\rho\tilde{D}^{hg}_{\rho\sigma}(y-t)\tilde{D}^{mn}_G(y-w) = \int d^d[uv]\tilde{\Gamma}^{(0)hbm}_\rho(u;y,v)\tilde{D}^{hg}_{\rho\sigma}(u-t)\tilde{D}^{mn}_G(v-w)\,. \tag{A18}$$

Using this, one can remove the spacetime derivatives in favor of bare ghost-gluon vertices and finally arrive at the complete DSE for the ghost-gluon vertex in position space:

$$\begin{aligned}\tilde{\Gamma}^{abc}_\mu(x;y,z) \;=\; & \tilde{\Gamma}^{(0)abc}_\mu(x;y,z)\\ & +\int d^d[rstuvw]\tilde{D}^{mn}_G(v-w)\tilde{\Gamma}^{and}_\mu(x;w,r)\tilde{D}^{de}_G(r-s)\tilde{\Gamma}^{gec}_\sigma(t;s,z)\tilde{D}^{hg}_{\rho\sigma}(u-t)\tilde{\Gamma}^{(0)hbm}_\mu(u;y,v)\\ & +\int d^d[rstuvw]\tilde{D}^{mn}_{\rho\nu}(u-w)\tilde{\Gamma}^{and}_{\mu\nu\lambda}(x,w,r)\tilde{D}^{de}_{\lambda\sigma}(r-s)\tilde{\Gamma}^{egc}_\sigma(s;t,z)\tilde{D}^{hg}_G(v-t)\tilde{\Gamma}^{(0)mbh}_\rho(u;y,v)\\ & -\int d^d[tuvw]\tilde{D}^{gh}_G(v-t)\tilde{\Gamma}^{nagc}_{\sigma\mu}(w,x;t,z)\tilde{D}^{mn}_{\rho\sigma}(u-w)\tilde{\Gamma}^{(0)mbh}_\mu(u;y,v)\,.\end{aligned} \tag{A19}$$

From this equation, eq. (7) follows straightforwardly by Fourier transformation.

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